Boundary Layer Flow and Heat Transfer along an Infinite Porous Hot Horizontall Continuous Moving Adiabatic Plate by Means of the Natural Transformation Method

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ABSTRACT: The present problem deals with the study of two- dimensional laminar boundary layer flow of a viscous, incompressible fluid along an infinite, porous, hot, horizontall, adiabatic, continuous moving plate. The governing partial differential equations are converted in dimensionless quantities and are solved using Natural Transform Technique. The expressions for velocity field, temperature field, rate of heat transfer and skin-friction have been obtained. The influence of various physical parameters, such as Eckert number Ec, Prandtl number Pr, Grashoff number Gr, plate velocity α and heat source/sink parameters is extensively discussed with the help of graphs to show the physical aspects of the problem. It is found that these parameters significantly affect the flow and heat transfer.

Keywords: Boundary layer flow, moving adiabatic plate, laminar flow, heat transfer and natural transform.

I. INTRODUCTION

The study of steady and unsteady boundary layers is useful in several physical problems such as flow over blades of turbines and compressors, flow over a helicopter in translation motion, flow over aerodynamics surfaces of vehicles in manned flight, etc. In view of these applications, Sakiadis [10] has studied theoretically the boundary-layer on a continuous semi-infinite sheet moving steadily through an otherwise quiescent fluid environment. The boundary layer solutions of Sakiadis [10] resulted in a skin friction of about 30% higher than that of Blasius [6] for the flow past a stationary flat plate. Later the experimental and theoretical studies were made by Tsou et al. [11] and Abdelhafez [1].

A number of works are presently available that follow the pioneering classical work of Sakiadis [9], Tsou, Sparrow and Goldstein [11] and Crane [8]. In 1996, Chiam [7] considered the heat transfer problem with variable thermal conductivity in stagnation-point flow towards stretching sheet. Ahmad and Marwah [3,4] and Ahmad et al. [5] also studied the boundary layer flow with heat transfer for linear stretching plate and with variable thermal conductivity.

Vajravelu [11] had obtained the solution of boundary layer flow and heat transfer over a continuous porous surface moving in an oscillating stream. In solving such boundary value problems, Loonker et al. [9] applied the Natural Transform for the distribution and Boehmians spaces. The recent development for the use of Natural Transform in solving the problems of boundary layer on moving horizontal plate has been done by Agarwal et al. [2].

II. FORMULATION OF THE PROBLEM

Consider the steady boundary layer flow and heat transfer of a viscous incompressible fluid over a continuous moving flat plate in the presence of constant suction at the surface, constant free stream U_∞ and volumetric rate of heat generation (or absorption), when the plate is an adiabatic i.e. there is no exchange of heat between the plate and the fluid. The plate is moving in flow direction with constant velocity U_w i.e. along X-axis and Y^* - axis is normal to the plate.

The governing boundary layer flow equations of continuity, motion and energy for flow of an incompressible viscous fluid along adiabatic flat continuous moving plate are-

Equation of Continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \quad \Rightarrow \quad v^* = -v_0(\text{constant}),\ v_0 > 0 \qquad \text{... (1)}$$

Equation of Motion

$$\rho\left(-v_0\frac{\partial u^*}{\partial y^*}\right)=\mu\frac{\partial^2 u^*}{\partial y^{*2}} \quad ...(2)$$

Equation of Energy

$$\rho C_P\left(-v_0\frac{\partial T^*}{\partial y^*}\right)=\kappa\frac{\partial^2 T^*}{\partial y^{*2}}+\mu\left(\frac{\partial u^*}{\partial y^*}\right)^2+Q\left(T^*-T_\infty\right) \quad ...(3)$$

where u^*, v^* are the velocity components along X^*- axis and Y^*- axis respectively, ρ is the density, v_0 the cross-flow velocity, μ the coefficient of viscosity, C_p the specific heat at constant pressure, κ the thermal conductivity, Q the volumetric rate of heat generation (or absorption) and T_∞ the free stream temperature. The corresponding boundary conditions are

$$\left.\begin{array}{l} y^*=0 \ \ : u^*=U_w, v^*=-v_0, \dfrac{\partial T^*}{\partial y^*}=0 \\ y^*\to\infty : u^*\to U_\infty, T^*\to T_\infty \end{array}\right\} \quad \ldots(4)$$

III. METHOD OF SOLUTION

Introducing the following non- dimensional quantities-

$$\left.\begin{array}{l} y=y^*\dfrac{v_0}{\upsilon}, u=\dfrac{u^*}{U_\infty}, \theta=\dfrac{T^*-T_\infty}{T_w-T_\infty}, \alpha=\dfrac{U_w}{U_\infty}, v=\dfrac{v^*}{v_0} \\ \Pr=\dfrac{\mu C_p}{\kappa}, S=\dfrac{Qv^2}{\kappa v_0^2}, Ec=\dfrac{U_\infty^2}{C_p\left(T_w-T_\infty\right)} \end{array}\right\} \quad ...(5)$$

into the equation (2) and (3), we get

$$u''+u'=0 \quad \ldots(6)$$

$$\theta''+\Pr\theta'+S\theta=-Ec\Pr(u')^2 \quad \ldots(7)$$

where Pr is the Prandtl number, Ec the Eckert number,S the heat source parameter and dash denotes the differentiation with respect to y. The corresponding boundary conditions in non- dimensional form are

$$\left.\begin{array}{l} y=0: \ u=\alpha, \theta'=0 \\ y\to\infty: \ u\to 1, \theta\to 0 \end{array}\right\} \quad ...(8)$$

where α is the velocity of plate.

The equations (6) and (7) are ordinary linear second order differential equations with constant coefficients and solved under the boundary conditions (8) with the help of natural transformation.

$$\left(\frac{s^2}{c^2}+\frac{s}{c}\right)\bar{u}(s,c)-\left(\frac{s}{c^2}+\frac{1}{c}\right)u(0)-\frac{1}{c}u'(0)=0$$

Now applying boundary condition at zero, we get

$$\bar{u}(s,c)=\frac{\alpha}{s+c}+\left\{u'(0)+\alpha\right\}\left(\frac{1}{s}-\frac{1}{s+c}\right)$$

Taking inverse Natural Transform,

$$u(y)=\alpha\left\{e^{-y}\right\}+\left\{u'(0)+\alpha\right\}\left[1-e^{-y}\right]$$

Boundary condition at infinite gives $u'(0)=(1-\alpha)$

Therefore,

$$u(y)=1+\left(\alpha-1\right)e^{-y} \quad ...(9)$$

Similarly, taking Natural Transformation to both side of (7), we get

$$\left(\frac{s^2}{c^2}+\Pr\frac{s}{c}+S\right)\bar{\theta}(s,c)-\left(\frac{s}{c^2}+\frac{\Pr}{c}\right)\theta(0)-\frac{1}{c}\theta'(0)=-\frac{Ec\Pr(\alpha-1)^2}{s+2c}$$

Boundary condition at zero, gives

$$\bar{\theta}(s,c)=\frac{K}{s+2c}+\{\theta(0)-K\}\frac{\left(s+\frac{c\Pr}{2}\right)}{\left(s+\frac{c\Pr}{2}\right)^2-\left(\frac{c^2\Pr^2}{4}-c^2S\right)}$$

$$+\frac{c\left\{-\left(\theta(0)-K\right)\frac{\Pr}{2}+K(2-\Pr)+\Pr\theta(0)\right\}}{\left(s+\frac{c\Pr}{2}\right)^2-\left(\frac{c^2\Pr^2}{4}-c^2S\right)}$$

Where $K=-\frac{Ec\Pr(\alpha-1)^2}{(4-2\Pr+S)}$. Applying Inverse Natural Transform, we get

$$\theta(y)=Ke^{-2y}+e^{\left(-\frac{\Pr}{2}y+\frac{y}{2}\sqrt{\Pr^2-4S}\right)}\left\{\frac{\theta(0)-K}{2}+\frac{\left(\theta(0)-K\right)\frac{\Pr}{2}+2K}{\sqrt{\Pr^2-4S}}\right\}$$

$$+e^{\left(-\frac{\Pr}{2}y-\frac{y}{2}\sqrt{\Pr^2-4s}\right)}\left\{\frac{\theta(0)-K}{2}-\frac{\left(\theta(0)-K\right)\frac{\Pr}{2}+2K}{\sqrt{\Pr^2-4S}}\right\}$$

Boundary condition at infinite gives

$$\theta(0)=\left(\frac{\Pr+\sqrt{\Pr^2-4S}-4}{\Pr+\sqrt{\Pr^2-4S}}\right)K$$

Therefore,

$$\theta(y)=Ke^{-2y}+\{\theta(0)-K\}e^{\left(\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}\right)y}$$

If $\theta(0)-K=-\frac{2Ec\Pr(\alpha-1)^2}{m_2(4-2\Pr+S)}=\frac{2}{m_2}K$ (say)

Where $K=-\frac{Ec\Pr(\alpha-1)^2}{(4-2\Pr+S)}$, $m_2=\frac{-\Pr-\sqrt{\Pr^2-4S}}{2}$ then above equation reduces to

$$\theta(y)=Ke^{-2y}+\frac{2}{m_2}Ke^{m_2y} \qquad \text{... (10)}$$

The equations (9) and (10) show the solution for velocity u(y) and heat transfer θ(y).

IV. SKIN-FRICTION COEFFICIENT

The skin-friction coefficient C_f at the plate is given by

$$C_f=\frac{\tau_w}{\rho U_\infty v_0}=\left(\frac{\partial u}{\partial y}\right)_{y^*=0} \quad \text{where } \tau_w=\mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0}$$

$$C_f=1-\alpha \qquad \text{... (11)}$$

Table 1. Numerical values of Skin friction C_f for different values of α.

Plate Velocity(α)	**Skin friction (C_f)**
0.0	1.0
0.5	0.5
1.0	0.0
1.5	-0.5
2.0	-1.0

V. NUSSELT NUMBER

The rate of heat transfer at the plate is zero (due to adiabatic system), which can be seen as follows-

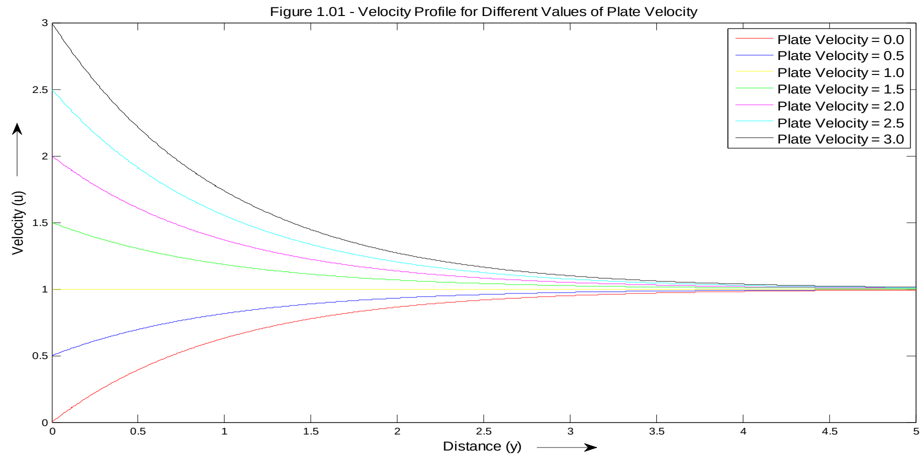
$$N_u = \frac{\upsilon q}{v_0 \kappa (T_w - T_\infty)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \quad \text{… (12)}$$

Hence the expression of the Nusselt number at the plate is given by

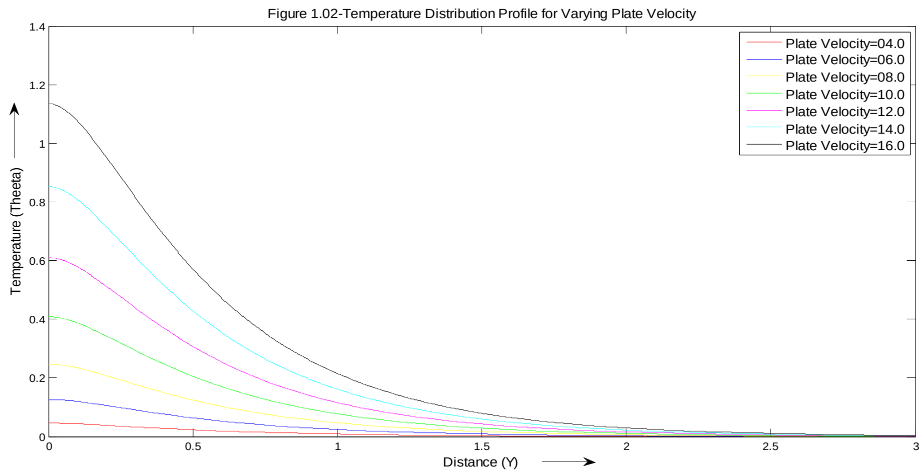
$$N_u = 2K - \frac{2}{m_2} m_2 K$$

$$N_u = 0 \quad \text{… (13)}$$

VI. RESULTS AND DISCUSSION

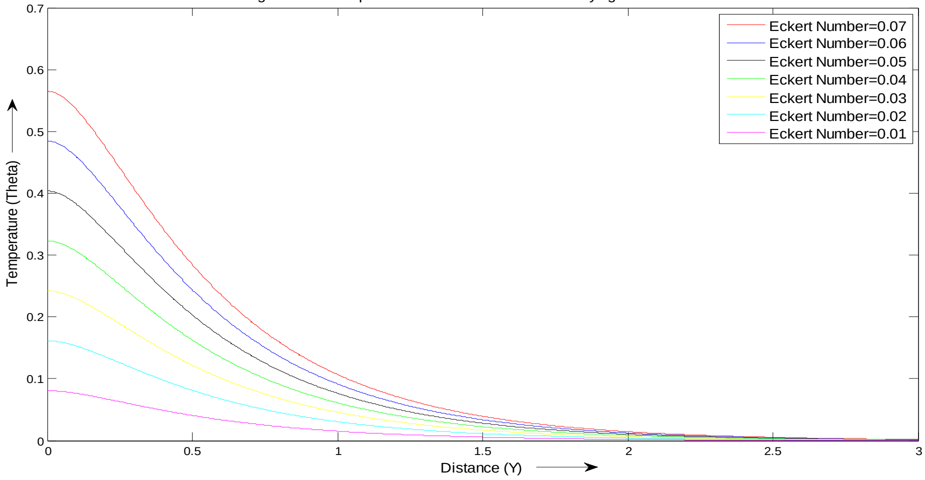


In this figure 1.01, the values of the mean velocity field u for various values of plate velocity α are summarized. It can be observed that u increases considerably as α increases. Moreover, when 0< α <1 (i.e. when the plate velocity is less than the free stream velocity), the profile for u is concave down, and when α >1(i.e. when the plate velocity is greater than the free stream velocity), the profile for u is concave up.



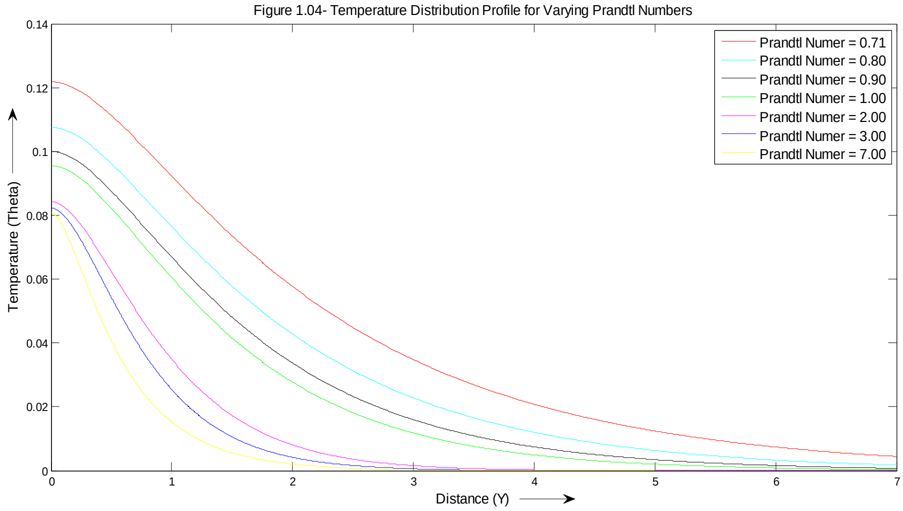
From the above figure 1.02 it may be observed that the fluid temperature increases with the increase of the plate velocity. We can also observe that in the very vicinity of the plate (y <0.01) curves appear to be convex above afterwards they became concave above.

Figure 1.03- Temperature Distribution Profile for Varying Eckert Number



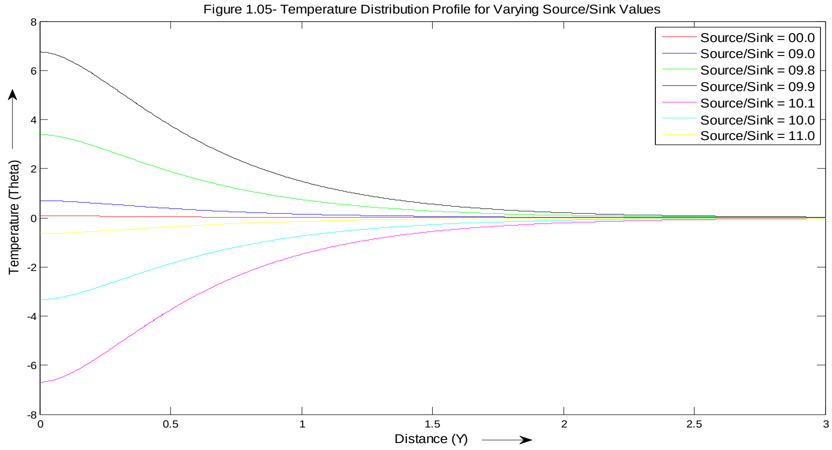
From this figure 1.03 we observe that the fluid temperature increases due to increase of Eckert Number, when the other parameters are kept fixed. Also we note that for a fixed Eckert Number, the fluid temperature θ decreases exponentially with the increasing values of distance y.

Figure 1.04- Temperature Distribution Profile for Varying Prandtl Numbers



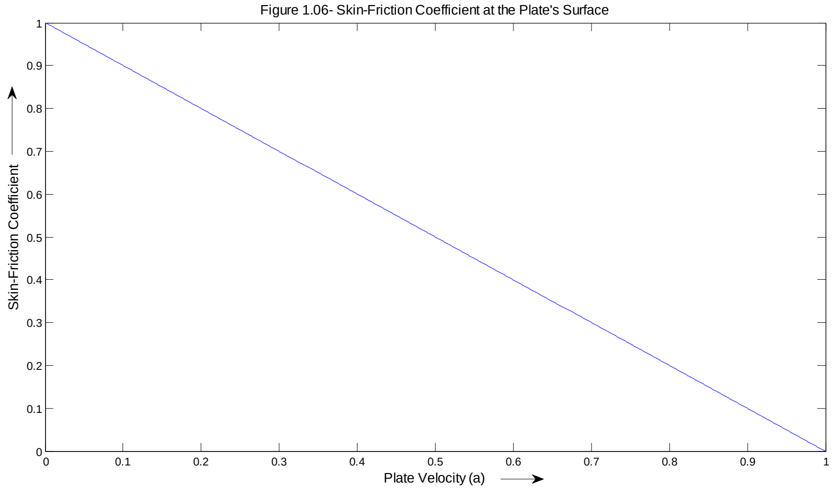
It can be noted from the figure 1.04 that the fluid temperature decreases due to increase of Prandtl number in the absence of heat source/sink parameter, when the plate is stationary or moving with constant velocity α. One can easily observe that in the very vicinity of the plate (y <0.01) curves appear to be convex above afterwards they became concave above.

Figure 1.05- Temperature Distribution Profile for Varying Source/Sink Values



From this figure 1.05, it can be observed that the fluid temperature increases with the increase of heat source parameter in sub-intervals like S=(9,10), (10, 11), while it may decrease from one interval to another.

Figure 1.06- Skin-Friction Coefficient at the Plate's Surface



The above figure 1.06 describes the behavior of the Skin-Friction at the plate. It is evident that Skin-Friction coefficient is a decreasing function of α. Physically it means that the Skin-Friction decreases for increasing values of the plate velocity keeping free stream velocity and other parameters as constant.

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